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A Conceptual Design Methodology to Predict the Wave Drag of a Transonic Wing

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Abstract

A conceptual design methodology to predict the wave drag of a transonic wing for use within multisdisciplinary aircraft design was developed. To achieve this, a database of cross section designs optimized with respect to total drag was set up varying the design parameters *Ma*, t/c, c_L and *Re*. Mathematical formulations for the aerodynamic cross section characteristics total drag, viscous drag and the local shock location were derived from the database as functions of the design parameters. The cross section wave drag was then derived using these formulations. A locally infinite swept wing is assumed and simple sweep theory using the shock sweep angle is used to transform the wave drag. The wave drag of a 3-D wing is predicted summing locally infinite swept wing sections in spanwise direction. The achieved drag prediciton is accurate enough for use within conceptual aircraft design and predicts well the trends in wave drag development as a function of the design parameters *Ma*, t/c, c_L, *Re* and the wing planform.

Introduction

Within the scope of multidisciplinary conceptual aircraft design [1] [2], a transonic aerodynamic module to predict transonic effects of a wing is essential. Typically, about 300 parameters and 50 design variables and constraints are used to describe the complete configuration, whereas about 150 parameters and 15 design variables of these are used for aerodynamic purposes [3].
The objective of the presented methodology is to describe transonic aerodynamic effects with a limited number of parameters and predict the correct trends within conceptual aircraft design. Processing time becomes critical for application of the methodology within multidisciplinary design automation and optimization. For this reason, time intensive codes (e.g. 3-D Navier Stokes, 3-D Euler) are not appropriate.
Therefor a fast conceptual design methodology to predict the wave drag of a transonic wing as a function of the design parameters *Ma*, t/c, c_L, *Re* and the wing planform will be described.

Methodology

Transonic aerodynamic characteristics of a wing are a result of the wing planform, the spanwise distribution of lift, thickness and local flow data. These parameters shall be used as input data for a method to predict aerodynamic effects of a transonic wing.
As a first step, a database of cross section designs was set up using numerical optimization routines. To achieve this, DaimlerChrysler Aerospace Airbus (DA) inhouse codes were linked to the Synaps optimization tool *PointerPro*^{TM} [4] to set up the database. The aerodynamic codes used were *VICWA* [5] (Full potential code with finite-difference boundary layer calculation) and *XLS6* (Full potential code with integral boundary layer calculation). The database contains about 240 cross section designs optimized with respect to total drag, called optimized performance designs. Typical optimized performance cross section designs of the database are shown in Fig. 1.

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All calculations are 2D, without sweep angle. The design parameters to set up the database of optimized performance cross section designs have been varied within the following ranges:

$$0.7 < Ma_{2d} < 0.8$$
$$0.5 < c_{L2d} < 0.9$$
$$0.05 < t/c_{2d} < 0.16$$

For a 3D wing the corresponding *Ma* range for quarter chord sweep angles of about 30° is $0.8 < Ma_{3d} < 0.95$. The Reynolds number *Re* was varied from $1 \cdot 10^6$ to $80 \cdot 10^6$.
Using this database of optimized performance cross section designs, analytical approximations for the relationships between aerodynamic cross section characteristics and the design parameters *Ma*, t/c, c_L and *Re* were derived from the database using appropriate methods. Formulas were found, ensuring physically plausible trends towards the borders of design parameter ranges.
The idea for a methodology to predict the 2D total profile drag as a function of the design parameters *Ma*, t/c, c_L at constant *Re* originates from A. Van der Velden.
A refined methodology to predict the 2D cross section total drag and a methodology for the profile viscous drag as a function of the design parameters *Ma*, t/c, c_L and *Re* were established.

Methodologies needed for the 2D wave drag:

- Total profile drag
- Profile viscous drag

To predict the wave drag of a transonic 3D wing, locally infinite swept wing sections are assumed which are integrated in spanwise direction. Simple sweep theory using the local shock sweep angle is used to transform the cross section wave drag. To achieve this, methodologies for the following aerodynamic characteristics as functions of the design parameters were developed.

Methodologies needed for the 3D wave drag:

- Profile wave drag
- Local shock sweep angle

The previously mentioned methods for the 2D total profile drag and the profile viscous drag were used to derive the profile wave drag. A methodology for the local shock sweep angle was developed in previous work [6].
The methodologies are used within multidisciplinary design automation to predict the minimum wave drag, the minimum total drag and trends in drag development of a transonic wing as a function of local values of *Ma*, c_L, t/c, *Re* and the wing planform.
The 3D design paramter ranges for the use of the methodologies are as follows:

$$0.7 < Ma < 0.95$$
$$0.1 < c_L < 1.3$$
$$0.03 < t/c < 0.18$$

Analysis

The total drag output from the transonic code of the optimized performance designs in the database was analyzed to determine the relationship between cross section characteristics and design parameters. A drag break down into wave drag due to lift and viscous effects is assumed to be adequately correct for use within conceptual design. Both wave drag due to lift and viscous drag are assumed to be functions of the design parameters *Ma*, t/c, c_L and Reynolds number *Re*.

Total profile drag

To describe the total profile drag of the optimized performance designs in the database, an appropriate function was chosen. As a first estimate a third order polynomial (1) was used with mixed terms in the design parameters *Ma*, c_L and t/c combined with an exponential contribution of *Ma*, c_L and t/c .

$$c_{D\,tot} = \sum_{\substack{i,j,k=0\\ i+j+k\le 3}}^{3} A_{ijk}\, x_1^i\, x_2^j\, x_3^k + \cdots$$

$$\cdots + \left(\sum_{i=0}^{3} x_i\right) \cdot exp\left(\sum_{\substack{i,j,k=0\\ i+j+k\le 2}}^{2} B_{ijk}\, x_1^i\, x_2^j\, x_3^k\right) \quad (1)$$

where

$$x_1 = f(Ma)$$
$$x_2 = f(c_L)$$
$$x_3 = f(t/c).$$

For better model handling, x_1, x_2 and x_3 are linear functions of the design parameters *Ma*, c_L and t/c, respectively.
For a specific cross section n in the database, the local quadratic deviation E_{locn} of the total drag model compared to the total drag in the database is described in equation (2). Here $(c_D)_{totdb_n}$ and $(c_D)_{totmod_n}$ are the total drag of the cross section n in the database and of the model following equation (1) respectively.

$$E_{locn} = (|(c_D)_{totdb_n} - (c_D)_{totmod_n}|)^2 \quad (2)$$

The mean global deviation E_{glob} of the mathematical model for the total drag for all N cross sections in the database may then be derived using equation (3).

$$E_{glob} = \frac{1}{N}\sum_{n=1}^{N} E_{locn} \quad (3)$$

To achieve the best fit possible for the model with the database, the coefficients A_{ijk} and B_{ijk} were varied until the minimum of the global deviation E_{glob} was reached. Due to the resulting nonlinear equation system, a numeric optimizer was used to find the coefficients A_{ijk} and B_{ijk} in equation (1).
Primarily a physically correct reproduction of trends in total drag development was intended. The mean deviation of the model compared to the total drag of the cross sections in the database is less than 7.5%. An exact agreement of the total drag determined from the database with the total drag calculated using equation (1) was not required for multidisciplinary design.
The total drag of the optimized cross sections in the database compared to the total drag model for constant Reynolds number at $c_L = 0.5$ is shown in Fig. 2. The total drag coefficient $(c_D)_{tot}$ is plotted as a function of t/c. The Mach number is a curve parameter and is increasing from bottom right to top left. The database total drag is shown as dashed lines and the model total drag as solid lines. Similar plots at $c_L = 0.7$ and $c_L = 0.9$ are shown in Fig. 3 and Fig. 4 respectively.
The finally derived formula of the total drag model principally consists of an absolute term, linear terms in c_L and t/c and functions of $(t/c)^2$. Third order terms in the polynomial contribution were not relevant. An exponential function with linear terms in *Ma* and c_L and a quadratic term in t/c was used to describe the total cross section drag for a specific Reynolds number.

$$c_{D_{tot}} = P_{tot}(c_L,\ t/c,\ (t/c)^2) + \cdots$$
$$\cdots + E_{tot}(Ma,\ c_L,\ (t/c)^2) \quad (4)$$

with
P_{tot} polynomial
E_{tot} exponential function

The total drag model for constant Reynolds number at $c_L = 0.5$ is shown in Fig. 5. The total drag coefficient $(c_D)_{tot}$ is plotted as a function of t/c and Mach number *Ma*. Mach number is increasing from front to back and thickness is increasing from left to right.
The total drag model predicts the correct trends in total drag development as a function of the design parameters *Ma*, t/c, c_L and Reynolds number *Re*. The behaviour near the borders of the parameter ranges is physically plausible.
In order to extract the profile wave drag from the profile total drag, the profile viscous drag as a function of the design parameters was needed.

Profile viscous drag

As for the total drag model a similar approach for the viscous drag model was used to find an analytical relationship of the viscous drag dependent on the design parameters t/c, c_L and *Re*. The optimized performance cross section designs in the database were used for the calculation of viscous drag at Reynolds numbers in the range from $Re = 1 \cdot 10^6$ to $Re = 80 \cdot 10^6$.
A second order polynomial with mixed terms in the design parameters c_L and t/c was used as a starting point for a specific Reynolds number.

$$c_{D visc}|_{Re} = \sum_{\substack{j,k=0 \\ j+k\leq 2}}^{2} A_{jk}\, x_2^j\, x_3^k \tag{5}$$

Again, the coefficients of the mathematical function to describe the viscous drag were found using numerical optimizers. The viscous drag model describing the viscous drag of the cross section designs in the database as a function of the design parameters is shown in equation (6). Note there is no contribution of Mach number to the profile viscous drag.

$$c_{D visc} = P_{visc}(c_L,\ t/c,\ (t/c)^2) \tag{6}$$

with

$$P_{visc} \quad \text{polynomial}$$

The coefficients A_{jk} in equation (5) are functions of the Reynolds number Re. Logarithmic terms or combinations of logarithmic and exponential terms are used, dependent on the coefficient A_{jk}.

$$\begin{aligned} A_{00} &= s_0 \quad e^{(\ln s_1 Re)} \\ A_{01} &= q_0 + \ln(q_1 Re) \\ A_{10} &= r_0 + \ln(r_1 Re) \\ A_{02} &= p_0 + p_1 Re \end{aligned} \tag{7}$$

The parameters p_i, q_i, r_i and s_i in (7) were calculated as previously described using a numerical optimizer to find a best possible fit for the viscous drag derived from the database. The Reynolds numbers were in the range from $Re = 1 \cdot 10^6$ to $Re = 80 \cdot 10^6$.
The viscous drag of the optimized cross sections in the database compared to the viscous drag model for constant Reynolds number at $c_L = 0.5$ is shown in Fig. 6. The viscous drag coefficient $(c_D)_{visc}$ is plotted as a function of t/c. The Mach number is a curve parameter and there is no variation of viscous drag with Mach number. The database viscous drag is shown as dashed lines and the model viscous drag as solid lines. A similar plot at $c_L = 0.7$ is shown in Fig. 7.
The viscous drag model for constant Reynolds number at $c_L = 0.5$ is shown in Fig. 8. The viscous drag coefficient $(c_D)_{visc}$ is plotted as a function of t/c and Mach number Ma. Mach number is increasing from front to back and thickness is increasing from left to right.
The viscous drag model predicts the correct trends in viscous drag development as a function of the design parameters c_L, t/c and Re. Additional drag due to possible separation at the upper surface trailing edge at off-design conditions (towards higher t/c at high c_L) is not described with the viscous drag model (Fig. 9). However, these conditions are not relevant for transonic aircraft sizing.

Profile wave drag

Using the profile total drag model and the profile viscous drag model described in the previous sections, a wave drag model can be established using the assumption about drag break down in the section 'Analysis'. Subtracting the viscous drag model at a certain Reynolds number from the total drag model set up for the same Reynolds number, the resulting model describes the wave drag contribution of the optimized cross section designs in the database as a function of Ma, t/c, c_L and Re. The wave drag model comprises the wave drag due to shock waves and pressure drag due to thickening of the boundary layer caused by shock boundary layer interaction.
Combining the wave drag model with the viscous drag model, the total drag of the cross sections can be predicted as a function of the design parameters Ma, t/c, c_L and Re.
The wave drag model for constant Reynolds number at $c_L = 0.5$ is shown in Fig. 10. The wave drag coefficient $(c_D)_{wave}$ is plotted as a function of t/c and Mach number Ma. Mach number is increasing from front to back and thickness is increasing from left to right.

Wing wave drag

Based on the assumption of an isobaric wing design concept, the wave drag can locally be calculated using simple sweep theory based on the local shock sweep angle (Fig. 11). The 3D transonic characteristics of a wing may then be predicted using the 2D models, derived from the database. The wing wave drag can be described as a function of the local values of Ma, c_L, t/c, Re, the local shock sweep angle ϑ_S and the wing planform.

The local shock sweep angle is a function of the local shock location of the upper surface shock and the wing planform. An analytical model of the local shock location as a function of the design parameters was derived from the database simimlar to the drag models described previously. The shock location of the upper surface shock of the optimized cross sections in the database compared to the shock location model at $c_L = 0.7$ is shown in Fig. 12. The shock location $(x/c)_s$ is plotted as a function of t/c. The Mach number is a curve parameter and is increasing from bottom right to top left. The database shock location is shown as dashed lines and the model shock location as solid lines. The shock location model at $c_L = 0.7$ is shown in Fig. 13. The shock location $(x/c)_s$ is plotted as a function of t/c and Mach number *Ma*. Mach number is increasing from front to back and thickness is increasing from left to right. More detailed information about the shock location model may be found in [6].

Using the viscous drag model, the wave drag model and the methodology described within this paper, the minimum total drag of a 3D wing for a specific design may be predicted and a drag break down into wing viscous drag and wing wave drag is possible.

The wing drag prediction solely is a function of the local design parameters *Ma*, c_L, t/c, *Re* and the wing planform. With the drag models being derived from cross section designs optimized with respect to minimum total drag, the methodology predicts the minimum possible drag for a specific wing design. With the assumption of an isobaric wing design concept, a good prediction for the outboard wing and a first good estimate for the inboard wing may be achieved using the methodology. However, the more an isobaric wing design concept is realized in actual wing design, the better the wing drag prediction which can be achieved using the described methodology.

Conclusion

A conceptual design methodology was described to predict the wave drag, viscous drag and total drag of a transonic wing. The methodology is intended for use within multidisciplinary conceptual design. The wave drag and trends in wave drag development of a wing can be predicted as a function of the local design parameters *Ma*, c_L, t/c, *Re* and the wing planform. As a result of the analytical description of the drag models used within the methodology, the processing time for a wing drag prediction is only in the order of seconds. Although the methodology is based on 2D cross section designs optimized with respect to minimum drag, the predicted minimum wave drag of a 3D wing compares well with the wing wave drag of current design studies. Future work will consider the implementation of effects of the fuselage and engine installation.

References

[1] Van der Velden, A., Aerodynamic Shape Optimization, AGARD-R-803 AGARD-FDP-VK1 Special Course, April 1994

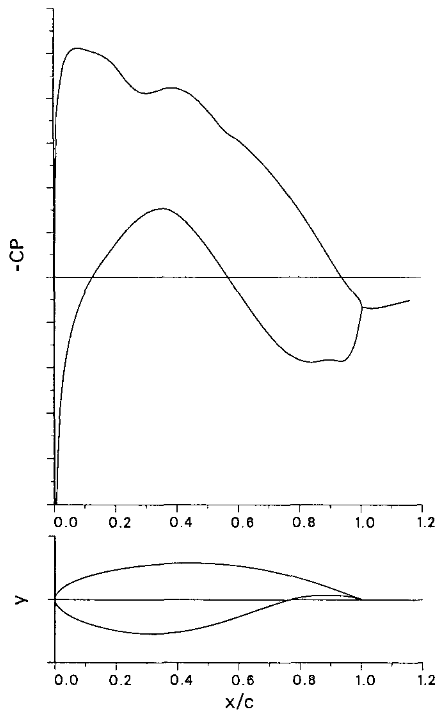
[2] Van der Velden, A., Tools for Applied Engineering Optimization, AGARD-R-803 AGARD-FDP-VK1 Special Course, April 1994

[3] Sobieczky, H., Seebass, R., Mertens, J., Dulikravich, G., Van der Velden, A., New design concepts for high speed air transport, CISM courses and lectures no. 366, International Center for Mechanical Sciences, Springer, 1997

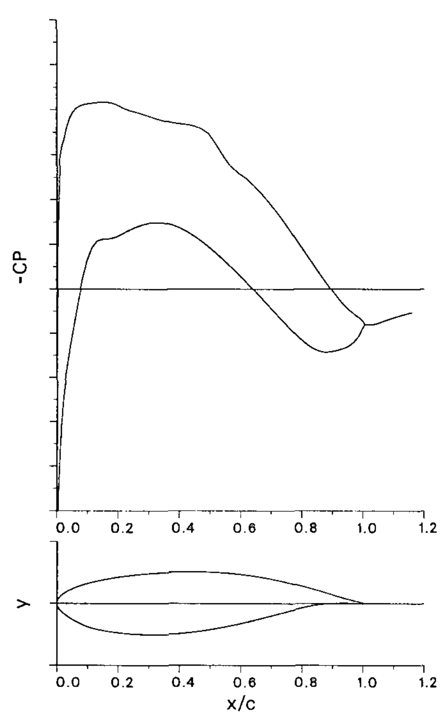
[4] Synaps Inc., "PointerPro^{TM}4.2", www.synaps-ing.de

[5] Dargel, G., Ein Programmsystem für die Berechnung transsonischer Profil- und konischer Flügelströmungen auf der Basis gekoppelter Potential- und Grenzschichtlösungen, DGLR Bericht 92-07, Köln, 1992

[6] Krißler, T., A Conceptual Design Methodology for the Estimation of the Local Shock Locations on Optimized Transonic Wings, DGLR AG-STAB Symposium, Berlin, 1998



(a) Optimized performance design, $Ma = 0.72$, $c_L = 0.7$, $(t/c) = 0.11$



(b) Optimized performance design, $Ma = 0.76$, $c_L = 0.5$, $(t/c) = 0.10$

Fig. 1: Typical cross section designs of the database

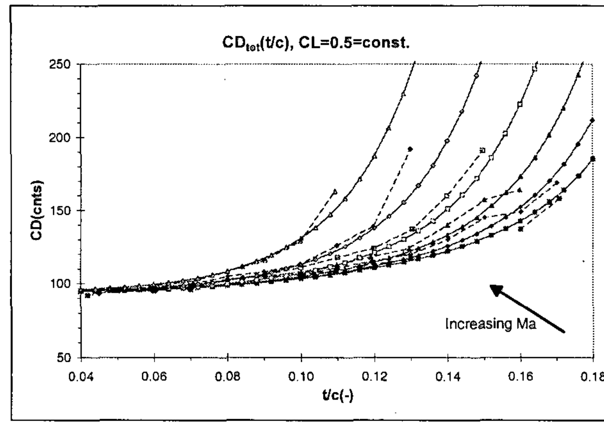


Fig. 2: Database total drag $c_{D_{totdb}}$ versus formulation $c_{D_{totmod}}(Ma,\ c_L,\ t/c)$ at $c_L = 0.5$

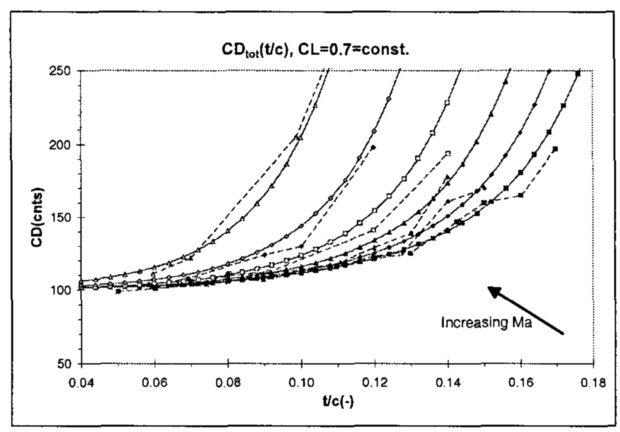


Fig. 3: Database total drag $c_{D_{totdb}}$ versus formulation $c_{D_{totmod}}(Ma,\ c_L,\ t/c)$ at $c_L = 0.7$

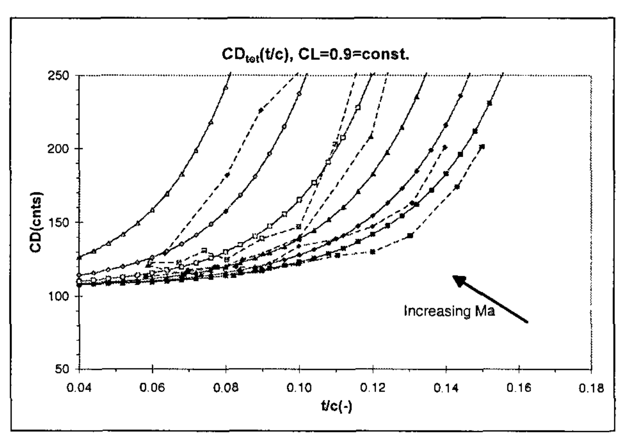


Fig. 4: Database total drag $c_{D_{totdb}}$ versus formulation $c_{D_{totmod}}(Ma,\ c_L,\ t/c)$ at $c_L = 0.9$

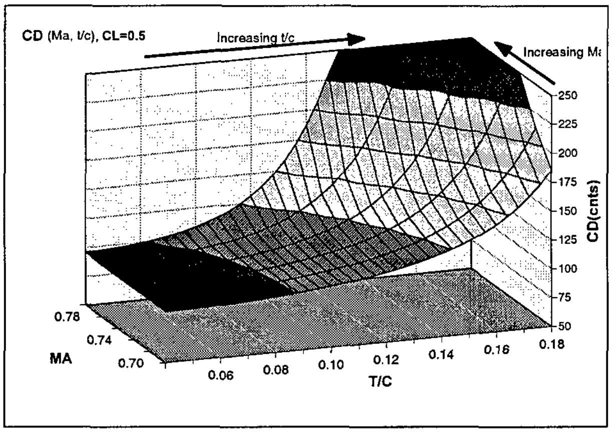


Fig. 5: Total drag $c_{D_{totmod}}$ at $c_L = 0.5$

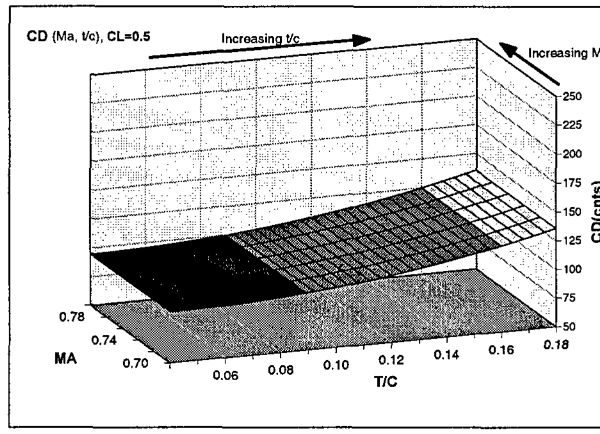


Fig. 8: Viscous drag $c_{D_{viscmod}}$ at $c_L = 0.5$

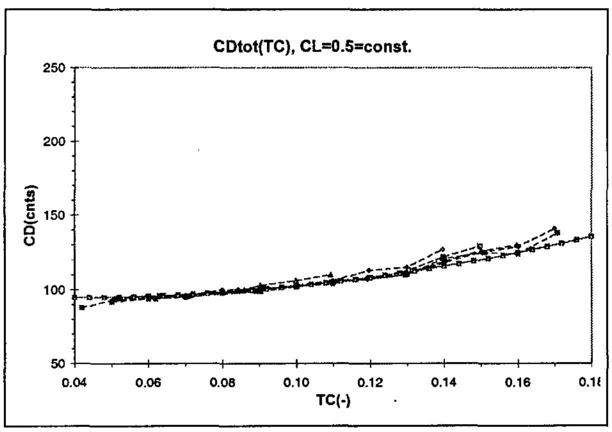


Fig. 6: Database viscous drag $c_{D_{viscdb}}$ versus formulation $c_{D_{viscmod}}(c_L,\ t/c)$ at $c_L = 0.5$

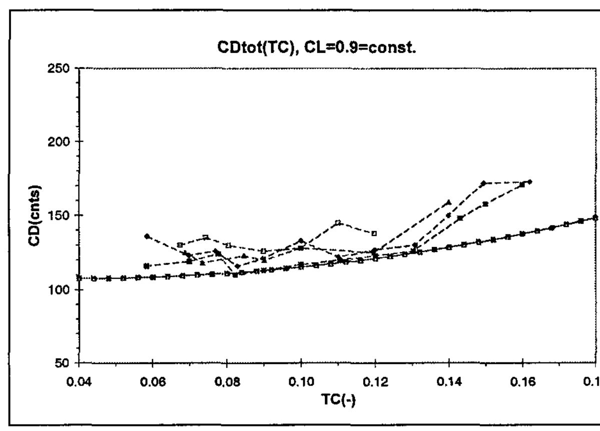


Fig. 9: Database viscous drag $c_{D_{viscdb}}$ versus formulation $c_{D_{viscmod}}(c_L,\ t/c)$ at $c_L = 0.9$

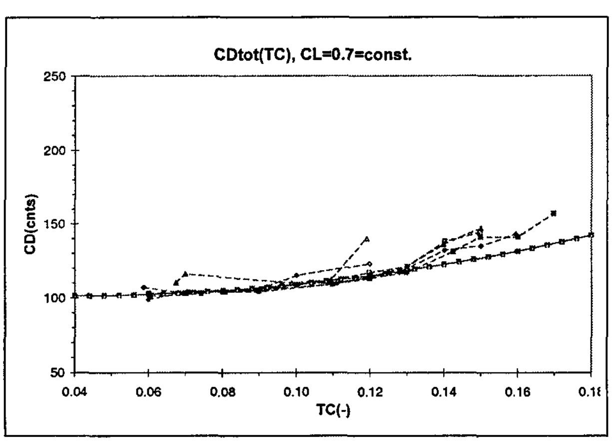


Fig. 7: Database viscous drag $c_{D_{viscdb}}$ versus formulation $c_{D_{viscmod}}(c_L,\ t/c)$ at $c_L = 0.7$

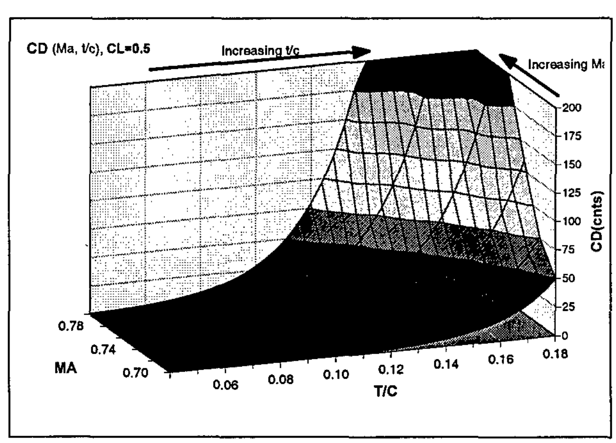


Fig. 10: Wave drag $c_{D_{wavemod}}$ at $c_L = 0.5$

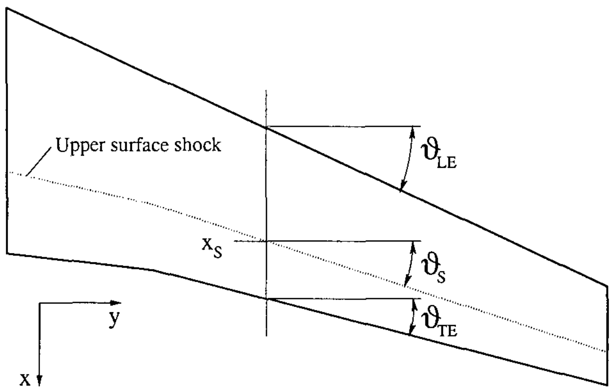


Fig. 11: Local shock sweep angle, isobaric wing design concept

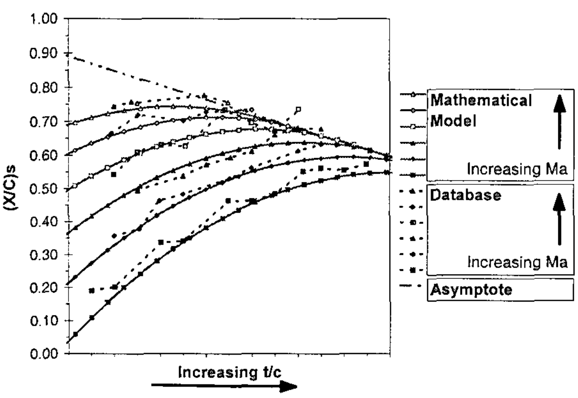


Fig. 12: Database shock location $(x/c)_{S_{db}}$ versus mathematical model $(x/c)_{S_{mod}}(Ma,\ t/c)$ at $c_L = 0.7$

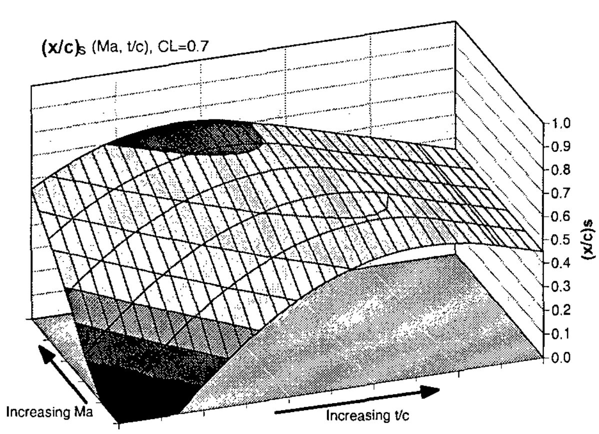


Fig. 13: Shock location model $(x/c)_{S_{mod}}(Ma,\ t/c)$ at $c_L = 0.7$